likely to have a basic grasp of the notation, including square and cube roots, and know that, e.g. $\sqrt{16}$ = 4 because 4^2 = 16 and $\sqrt[3]{8}$ = 2 because 2^3 = 8.

Students should recognise that the square (or cube) root of any number can be found, but that it is only when they are perfect square (or cube) numbers that this operation will give an integer solution.

In Key Stage 3, students will need to explore positive integer exponents greater than three. This will support other Key Stage 3 work involving writing numbers as the product of prime factors in simplified terms, thus enabling identification of the highest common factor and the lowest common multiple of two or more positive integers.

Key ideas

- Understand the concept of square and cube
- Understand the concept of square root and cube root
- Understand and use correct notation for positive integer exponents
- Understand how to use the keys for squares and other powers and square root on a calculator

Understand and use the unique prime factorisation of a number

Finding factors of a number will be familiar from Key Stage 2. Students should be able to find factor pairs for a given number and know that a number which has exactly two factors is prime. Students are expected to recall prime numbers up to 19 and be able to establish prime numbers up to 100. The focus in this set of key ideas is to be able to identify factors and prime numbers based on a deep understanding of number structure. Where rules for divisibility are used to help these processes, the focus should be on understanding why these rules work.

Students' experience of highest common factors and multiples at Key Stage 2 is likely to be limited to their work on simplifying fractions and checking to see if they have found the greatest number that is a factor of both the numerator and denominator. Similarly, when expressing fractions in the same denomination in order to compare them, for example, students may have identified the least common multiple of the two denominators even if this formal term has not been used.

In Key Stage 3, students will come across the unique prime factorisation property for the first time. Students will need to recognise that any positive integer greater than one is either a prime number itself or can be expressed as a product of prime numbers, and that there is only one way of writing a number in this way. It is this property that will help students to identify efficiently the highest common factor and lowest common multiple for two or more positive integers.

Key ideas

- Understand what a factor is and be able to identify factors of positive integers
- Understand what a prime number is and be able to identify prime numbers

- Understand that a positive integer can be written uniquely as a product of its prime factors
- Use the prime factorisation of two or more positive integers to efficiently identify the highest common factor*
- Use the prime factorisation of two or more positive integers to efficiently find their lowest common multiple

Exemplified significant key ideas

Identify and explain whether a number is or is not a multiple of a given integer

Common difficulties and misconceptions: students often find multiples of an integer by listing numbers in the specified times table. This strategy is efficient for small numbers of multiples but can lead to misconceptions, such as thinking that numbers have only 12 multiples or that numbers outside of the times tables do not have multiples.

Students need to be able to identify the patterns present in multiples of an integer and explore the structures which generate those patterns. For example, students should understand that adding two different multiples of the same number results in another multiple of that number. Similarly, that if a number is a multiple of 15, for example, it is also a multiple of five and of three. By exploring multiples and reasoning in this way, students can decide whether any number is or is not a multiple of a given integer.

Strategies for identifying multiples usually link to division, especially for larger numbers which are not multiples known from multiplication tables. Students who find it challenging to make the connection between the idea of multiples (numbers in multiplication tables) and division may benefit from revisiting prior work on $factor_a \times factor_b = product$, and variations of this: $product \div factor_a = factor_b$.

The use of partitioning can also be a useful strategy when identifying multiples. For example, 6 132 is not a multiple of eight because 6 132 = 6 000 + 120 + 12 and while 6 000 and 120 are both multiples of eight, 12 is not. Using divisibility rules to test whether a number is a multiple or not may also be helpful. If using these, students should be given time to investigate both why they work and how they can be used.

Students should also understand the connections between multiplication tables. For example, students should know that all multiples of ten are multiples of five but not all multiples of five are multiples of ten. The use of a multiplication grid may support students to see these connections, consider the structures behind them and, consequently, be able to reason fully.

Students may have only experienced multiples as a list of positive integers. Defining multiples by a generalised statement, such as, *'For any integers a and b, a is a multiple of b if a third integer c exists so that a = bc'* will help students understand that 14, 49, 70 and −21 are all multiples of seven because 14 = 7 × 2, 49 = 7 × 7, 70 = 7 × 10 and −21 = 7 × −3. Examples are given below.

Example 1:

a) *Place a tick (✓) in the cell if the number is a multiple of 2, 5 or 10. Explain how you know.*

	2	**5**	**10**
830			
457			
12 974			
60 535			
519 276			

b) *What general statements can you make about multiples of 2, 5 and 10?*

c) *Find an integer which is a common multiple of 2, 5 and 10. Can you find another? And another? What do you notice?*

Example 1 has been designed to draw students' attention to patterns in multiplication tables. Students should be familiar with their two, five and ten multiplication tables, so this example gives an opportunity to explore the relationships between the multiples and the structures behind them.

Asking students to explain their reasoning in many different ways (for example, an integer is a multiple of two if: *'It is an even number'*, *'It has a 1s digit which is 0, 2, 4, 6, or 8'*, *'When halved, the quotient is an integer'*, etc.) can help students to generalise the structure of such numbers.

There is an opportunity to model generalised statements with precise language, such as:

'An integer is a multiple of two if…'

'An integer is a multiple of five if…'

'An integer is a multiple of ten if…'

Students can be supported to generalise this idea by offering the following sentence structure:

'a is a multiple of b if a third integer c exists so that a = bc.'

Similar questions can be used to explore the relationships between multiples of 2, 4 and 8 and multiples of 3, 6 and 9.

Example 2:

a) *Decide whether each statement is always, sometimes or never true.*

(i) *If a number is a multiple of 10, it is also a multiple of 5.*

(ii) *If a number is a multiple of 4, it is also a multiple of 8.*

(iii) *If a number is a multiple of 9, it is also a multiple of 2.*

(iv) *Multiples are positive integers.*

b) *Is it always, sometimes or never true that adding two consecutive multiples of 5 will give a multiple of 10?*

c) *Is it always, sometimes or never true that adding five consecutive multiples of 2 will give a multiple of 10?*

Students who have demonstrated a secure understanding of identifying multiples should be encouraged to go deeper by solving more complex problems, such as exploring all possibilities, creating their own examples and testing conjectures. Students should solve problems where there is more than one answer and there are elements of experimentation, investigation, checking, reasoning, proof, etc.

It is important that students are given opportunities to investigate multiplicative relationships. *Example 2* provides a structure for this as well as an opportunity to reason and use examples and counter-examples to demonstrate their mathematical understanding.

Example 3: *Two lighthouses flash at different intervals. One flashes every 5 seconds and the other every 8 seconds.*

At exactly midnight (00:00:00) they flash together. When will they next flash at the same time?

Example 3 provides students with an opportunity to apply their mathematical understanding to unfamiliar problems and to practise their understanding of a concept (i.e. intelligent practice rather than mechanical repetition) through focusing on relationships, rather than the procedure.

Use the prime factorisation of two or more positive integers to efficiently identify the highest common factor

Common difficulties and misconceptions: fundamental to this concept is that students are at ease with the idea that 2 × 3 × 5 is just another way of expressing the number 30 and does not need to be calculated. In fact, by leaving it in this form, it is much easier to discern factors and (when there are two numbers expressed in this way) to discern their common factors. However, students often struggle with this idea. When asked whether 2 × 3 × 5 is a multiple of 10 or not, it is not uncommon for students to multiply the three factors together to obtain 30 before they are able to say that it is a multiple of 10.

An important awareness in this key idea is that when two numbers are written as the product of two or more factors, looking for overlaps between the two products helps to find a common factor. For example, by writing 30 = 5 × 6 and 105 = 5 × 21, it is easily seen that five is a common factor.