Year 7 spring term

Plotting coordinates

Overview

In Key Stage 2, students should have become familiar with coordinates in all four quadrants. They should have made links with their work in geometry by both plotting points to form common 2D quadrilaterals and 'predicting missing coordinates using the properties of shapes' (Department for Education, 2013). These skills are developed further in Key Stage 3. A key focus will be thinking about x- and y-coordinates as the input and output respectively of a function or rule, and appreciating that the set of coordinates generated and the line joining them can be thought of as a graphical representation of that function.

Later in Key Stage 3, significant attention will be given to exploring linear relationships and their representation as straight line graphs. Students should appreciate that all linear relationships have certain key characteristics:

- a specific pair of values or points on the graph; for example, where $x = 0$ (the intercept)
- a rate of change of one variable in relation to the other variable; for example, how the y-value increases (or decreases) as the x-value increases (the gradient).

Students should be able to recognise these features, both in the written algebraic form of the relationship and in its graphical representation.

Prior learning

Before beginning graphical representations at Key Stage 3, students should already have a secure understanding of the following learning outcomes from study at upper Key Stage 2:

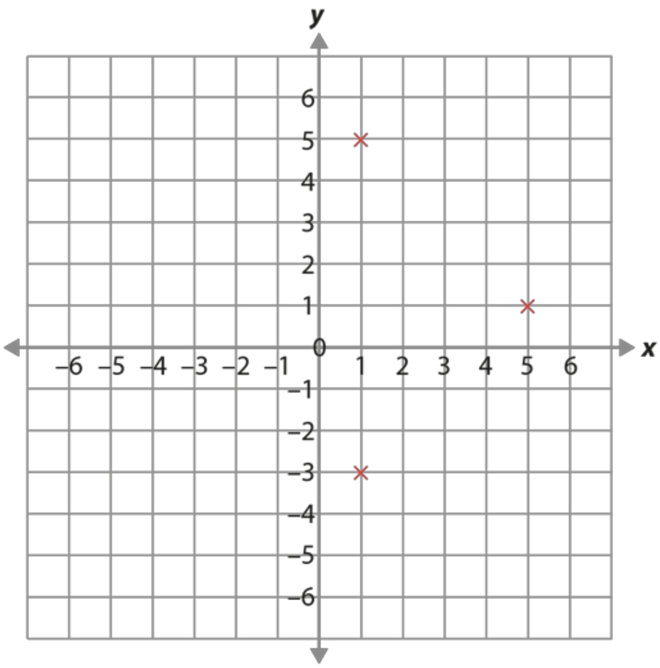
- Describe positions on the full coordinate grid (all four quadrants)
- Find pairs of numbers that satisfy an equation with two unknowns
- Enumerate possibilities of combinations of two variables

The NCETM has created the following Key Stage 2 ready-to-progress criteria to support teachers in making judgements about students' understanding and knowledge.

4G-1 Draw polygons, specified by coordinates in the first quadrant, and translate within the first quadrant.

Checking prior learning

The following activities from the Standards & Testing Agency's past mathematics papers offer useful ideas for teachers to use to check whether prior learning is secure:

Reference	Activity
2018 Key Stage 2 Mathematics paper 3: reasoning question 10	*Layla draws a **square** on this coordinate grid.* *Three of the vertices are marked.*  *What are the coordinates of the missing vertex?*

Language

Cartesian coordinate system, gradient, intercept, linear

Progression through key ideas

Connect coordinates, equations and graphs

Students should be fluent at both reading and plotting coordinates involving negative and non-integer *x*- and *y*-values in all four quadrants. They should be confident in solving problems that require them to be analytical, and be able to go beyond finding an answer to being able to give clear reasons based on the relationships between the coordinates (a key element in this core concept).

For example, in ‘Checking prior learning’ (above), in order to determine the coordinates of the missing vertex, students could:

- Identify the gradient of one of the sides of the square and infer the gradient of the opposite side.
- Use the fact that the diagonals of the square are perpendicular and of equal length.

A sound understanding of the relationships between the *x*- and *y*-values of pairs of coordinates provides the basis for more sophisticated analysis of the features of linear functions and their graphs, which students will need to develop throughout Key Stage 3.

By graphing sets of coordinates where the *x*- and *y*-values are connected by a rule, students will become aware of the connection between a rule expressed algebraically and the graph joining the set of points. Students will then also need to think about horizontal and vertical straight line graphs where the functions ($x = a$ and $y = b$) are of a particular form, and relate the concepts of gradient and intercept to these. This work should lead students to appreciate the important two-way connection, that is:

- If the *x*- and *y*-values of the coordinates fit an arithmetic rule, then they will lie on a straight line.
- If the coordinates lie on a straight line, then their *x*- and *y*-values will fit an arithmetic rule.

Key ideas

- Describe and plot coordinates, including non-integer values, in all four quadrants
- Solve a range of problems involving coordinates
- Know that a set of coordinates, constructed according to a mathematical rule, can be represented algebraically and graphically*
- Understand that a graphical representation shows all of the points (within a range) that satisfy a relationship

Exemplified significant key ideas

Know that a set of coordinates, constructed according to a mathematical rule, can be represented algebraically and graphically

Common difficulties and misconceptions*:* when working with linear equations and graphs, it is not uncommon for students to accept that integer coordinates fit the rule given by an equation. What students may not appreciate is that the line is representing an infinity of points, all of which fit the rule.

It will be important for students to experiment with coordinates in between integer points they have used to construct the line, and to verify that these coordinates also fit the rule.

This should lead students to the important awareness of the key idea that if a set of coordinates lies on the same straight line, then there is a consistent relationship between the *x*- and *y*-values that can be expressed algebraically as the equation of the line. Students should be encouraged to plot the coordinates themselves to confirm that the coordinates do, in fact, lie on a straight line; but also to think deeply about why this is so and not just rely on practical demonstration.

These more probing explorations will support students in reaching two important awarenesses:

- The line represents the infinity of points satisfying the rule and therefore 'captures' or represents that rule in the same way the algebraic equation does.
- The line divides the plane into points that fit the rule and points that do not.

Some students may find it challenging to express the relationship between the *x*- and *y*-values algebraically. Asking students to test a given algebraic relationship by generating another coordinate and testing whether this lies on the same straight line can help them to overcome this difficulty.

Encouraging the use of precise language will also help students to overcome difficulties; establishing the relationship and articulating it using key vocabulary will enable students to discuss and reason with clarity. Prompting students to describe the relationship in words by considering how the *x*-value is being operated on in order for it to match the *y*-value, will help students identify the relationship before formally expressing it in algebraic form.

Example 1: *For each set of coordinates:*

a) Find the relationship between the x- and y-values.
b) Can you draw a straight line which passes through them?

(i) (0, 2) (1, 3) (2, 4)	*(v) (0, −3) (5, 2) (−3, −6)*
(ii) (0, 3) (1, 4) (2, 5)	*(vi) (−2, −3.5) (½ ,−1) (9, 7.5)*
(iii) (0, 4) (1, 5) (2, 6)	*(iv) (−3, −3) (1, 1) (4, 4)*
(v) (−3, −1) (0, 2) (3, 5)	*(viii) (2, 6) (4, 7) (−1, −3)*

c) *Find another point on each line. Do the x- and y-values have the same relationship?*

In *Example 1*, students find the relationship between the *x*- and *y*-values in sets of coordinates where that relationship is additive.

In parts *(i)*, *(ii)*, *(iii) and (iv)*, the equations are in the form $y = x + c$. Parts *(i)*, *(ii) and (iii)* start with a coordinate that has an *x*-value of zero. Students might find this helpful when starting to identify the relationship between the *x*- and *y*-values. Part *(iv)* starts with a coordinate that has an *x*-value of −3 to test whether students can accurately identify the relationship without zero as a starting point.

Parts *(v)* and *(vi)* have equations in the form $y = x - c$. In part *(v)*, the first coordinate has an *x*-value of zero to allow students to more easily identify the relationship, but this is not the case in part *(vi)*, where both a fraction and decimals are used.

In part *(vii)*, the equation is $y = x$. This part provides an opportunity for students to explore whether this is the same as $y = x + 0$ or $y = x - 0$.

Part *(viii)* has no linear relationship. This part will help to assess whether students are testing the relationship with all the coordinates provided and may help them understand why this is necessary. Plotting the coordinates may support this understanding.

In all parts, a range of coordinates has been provided, including those with negative and fractional values. The order in which they have been written also varies, as students need to become adept at selecting the easiest coordinates to work with first before testing the relationship with the others. Using the first coordinate listed for each part might not be the most efficient option.

Encourage students to use precise language when describing the relationship between the *x*- and *y*-values. In part *(i)*, students may identify that the *y*-values and the *x*-values are both increasing by one. It is important, however, that students can verbalise the relationship between the *x*- and *y*-values and in multiple ways (i.e. that the *y*-value is two more than the *x*-value; *y* subtract two equals the *x*-value; two added to the *x*-value is always the *y*-value, etc.).

Example 2: *For each set of coordinates:*

a. *Find the relationship between the x- and y-values.*
b. *Can you draw a straight line which passes through them*

(i) (6, 12) (−2, −4) (0, 0)	*(ii) (4, 2) (−3, −1.5) (5, 2.5)*
(iii) (−1, −3) (7, 21) (4, 12)	*(iv) (3, −3) (−6, 6) (−1.5, 1.5)*
(v) (½, 2½) (−2, −10) (2, 10)	*(vi) (−3, 6) (2, −4) (0, 0)*

In *Example 2*, students find the relationship between *x*- and *y*-values in sets of coordinates where the relationship is multiplicative.

In parts *(i)*, *(ii) and (iii)*, the *x*-value is multiplied by a positive integer.

In part *(iv)*, the *x*-value is multiplied by 0.5 (or divided by two). This provides an opportunity to discuss the different ways in which the equation can be written.

In parts *(v)* and *(vi)*, the *x*-value is multiplied by a negative integer.

Example 3*:*

a) (−10, −2)   (−2, 6)   (6, 14)

Charlie thinks the equation of the line passing through these coordinates is x = y + 8.

Explain why Charlie is wrong.

b) (10, 2)   (1, 5)   (−3, −15)

Mia thinks the equation of the line passing through these coordinates is y = 5x.

Explain why Mia is wrong.

Example 3 gives students an opportunity to explore misconceptions when finding the relationship between *x*- and *y*-values in sets of coordinates.