Understand proportionality

As students' understanding of multiplicative relationships matures, they can work with more complex contexts and use algebraic notation to generalise. An important awareness here is that there is one unifying structure which connects fractions, percentages and ratio, and that this one structure can be described by the algebraic formulae $x \times k = y$ or alternatively $k = \frac{y}{x}$, where x and y are the quantities in proportion and k is the constant of proportionality. While exploring a wide range of examples of proportionality (including examples of *'what it's not'*) it will be important to make the distinction between linear relationships which are not proportional (i.e. of the form $y = mx + c$ rather than $y = kx$) and also to become aware of situations where the variables are inversely proportional (i.e. $y = k \times \frac{1}{x}$ or $y = \frac{k}{x}$).

In formalising this generalisation, students are able to use the underlying structure to develop an awareness that there are different types of proportionality, particularly inverse proportionality.

Key ideas

- Understand the connection between multiplicative relationships and direct proportion
- Recognise direct proportion and use in a range of contexts, including compound measures
- Recognise and use inverse proportionality in a range of contexts

Exemplified significant key ideas

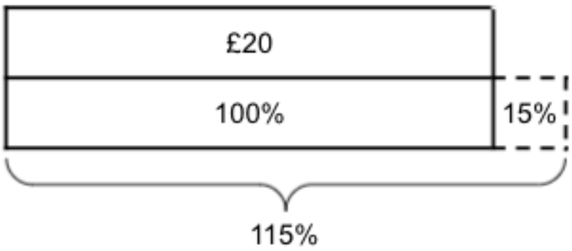
Calculate percentage changes (increases and decreases)

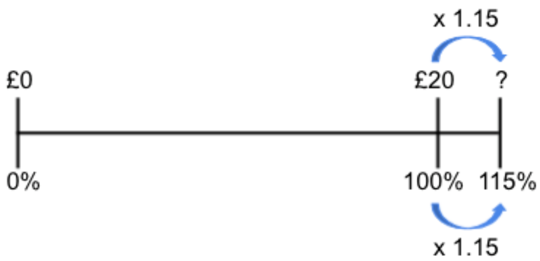
Common difficulties and misconceptions: students should be confident with using informal additive methods to increase or decrease an amount using a percentage. While it is important for students to be able to work flexibly with percentages, it is important for efficiency and depth of understanding that students understand there exists a single multiplier linked to any percentage change and recognise them as examples of multiplicative relationships.

Some students have difficulties using the additive method as they fail to find the final amount by adding or subtracting the increase/decrease to the original amount. Some students have difficulties with identifying the multiplier for single-digit percentages, such as 5%.

Bar models, double number lines and ratio tables are all powerful representations to help students work ‘beyond 100%’ and identify the both the whole, and the multiplier linked to the percentage.

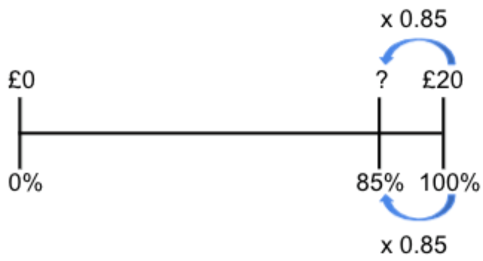
Increase £20 by 15%





£	%
20	100
?	115

Decrease £20 by 15%



£	%
20	100
?	85

Example 1: *Increase:*

a) *£30 by 50%*
   £50 by 30%

b) *£40 by 60%*
   £60 by 40%

c) *£100 by 10%*
   £10 by 100%

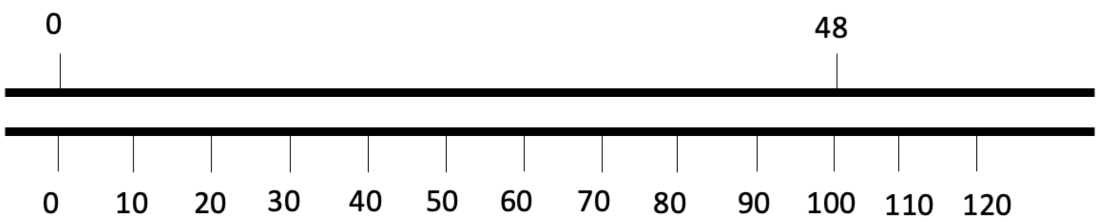
Here, students can be introduced to percentage changes using an additive approach before the use of a single multiplier to calculate the change. Recognising that the amount to be added on in each part of the question is the same, but that the final total is different and is dependent on the starting amount is a key step in understanding the multiplier.

Example 2: *Tick the calculations that correctly represent each statement:*

Statement	**Calculation**	**Tick**
Increase £20 by 35%	20 × 1.35	
Decrease £20 by 35%	20 × 0.35	
Increase £35 by 20%	35 × 0.2	
Decrease £35 by 20%	35 × 0.8	
Increase £45 by 1%	45 × 1.1	
Decrease £35 by 1%	45 × 0.99	

The questions in *Example 2* support students to recognise that there exists a single multiplier linked to any percentage change. Using multiplicative reasoning, students can increase a quantity by a percentage more efficiently than by using additive methods. Students need to be confident and fluent with recognising a single multiplier exists linked to any percentage change.

Example 3: *A chocolate bar that usually weighs 48 g has a special offer and now comes with 15% extra free.*



a) Mark the new weight on the double number line.
b) Use the double number line to estimate the new weight.

The same percentage increase can be calculated using this ratio table.

100	115
48	

c) Mark the multipliers on the ratio table.
d) Use the ratio table to calculate the new weight.

Example 3 offers an opportunity to connect ratio tables and double number lines to percentage changes.

If students are already familiar with these as representations of multiplicative relationships, then their use here will support them in connecting percentage changes with other situations which share the same mathematical structure, even though they may initially look different.

Part *b* asks students to use the double number line to estimate the new weight, but to use the ratio table to calculate. It might be argued that the double number line allows an insight into the structure, while the ratio table (which is an abstraction of the double number line, with the same information represented but without the scale) allows for easy identification of the multiplier and hence for calculation of the solution.