- Letter as generalised number: seen as being able to take several values rather than just one.
- Letter as variable: representing a range of unspecified values, and a systematic relationship is seen to exist between two sets of values.

The first three offer an indication of the difficulties and misconceptions students might have. The last three outline the progression that students need to make as they develop an increasingly sophisticated view of the way letters are used to represent number.

***Example 1:*** *For each of the following statements, use a letter to represent the number Isla is thinking of and write the statement using letters and numbers.*

*a) 'I am thinking of a number and I add three.'*
*b) 'I am thinking of a number and I multiply by two and add three.'*
*c) 'I am thinking of a number and I add three and multiply by two.'*
*d) 'I am thinking of a number and I multiply by three and add two.'*
*e) 'I am thinking of a number and I add two and multiply by three.'*

In *Example 1*, the numbers are deliberately kept the same in order for students to focus on the order of operations and how algebraic symbolism is used to represent the different order of operations, using brackets where necessary.

A key purpose of variation is to support students' awareness of what can change, and it can be useful to ask them to make up some examples like these for themselves. For example, through using activities such as: '*Using the numbers two and three, make up some different "I am thinking of a number" statements and set them for your partner.'*

While this example is a useful precursor to solving equations, the central purpose here is to understand that letters can have a range of values and to get a sense of how the value of expressions can change with these different values. Students should be encouraged to offer a number of possible values for $x$.

This is a good opportunity to introduce the language of 'variable' and encourage students to use this term while discussing their answers and their reasoning. For example*, 'In the expression 2x + 3, x is a variable because it can take a range of different values.'*

***Example 2:*** *For each of the following statements, use a letter to represent the number Isla is thinking of, write the statement using letters and numbers, and find the number she is thinking of.*

*a) 'I am thinking of a number; I add four and the answer is 12. What number am I thinking of?'*
*b) 'I am thinking of a number; I add four, multiply by three and the answer is 12. What number am I thinking of?'*
*c) 'I am thinking of a number; I add four, multiply by three, subtract six and the answer is 12. What number am I thinking of?'*

d) *'I am thinking of a number; I add four, multiply by three, divide by two and the answer is 12. What number am I thinking of?'*

The focus of *Example 2* is to make students aware of the fact that, when constraints are put on a situation, the unknown will take a particular value. The numbers have been chosen in *Example 2* to keep the given answer of 12 the same and to build the operations in sequence. The example will best be tackled by offering and discussing each part individually.

Students can be encouraged to make up their own examples for their partners. This will support their realisation that, when they put constraints on a situation like this, their partner will always be able to figure out their number.

Students should be encouraged to use the term 'specific unknown' when talking about these examples, as in, '*When I am told that 3(x + 4) – 6 =* 12*, there is only one value that will make this true and so the letter x stands for a specific unknown'.*

***Example 3:*** *Arrange these cards in order.*

| | | |
|---|---|---|
| $x$ | $2x$ | $x^2$ |
| $x + 3$ | $x^2 - 5$ | $3x - 2$ |

For *Example 3*, the cards could be given to six students and the students asked to line themselves up, holding their cards in front of them, in order for the whole class to engage with the problem collaboratively. As the statements on the cards are expressions involving variables, it is not possible to agree an order. This activity is intended to bring to the surface the students' current thinking (including misconceptions) and to engage them in discussion about the possible values these expressions can take.

## Understand that relationships can be generalised using algebraic statements

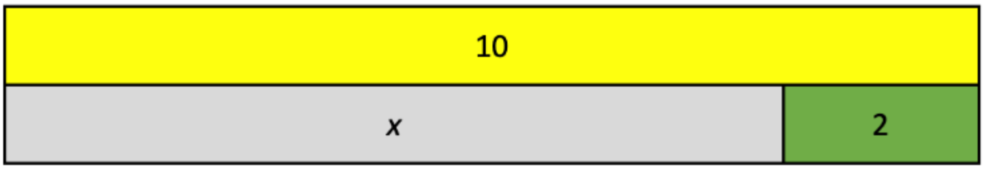
***Common difficulties and misconceptions****:* the non-statutory guidance for the Year 6 programme of study states that *'Students should be introduced to the use of symbols and letters to represent variables and unknowns in mathematical situations that they already understand.'* They may have some familiarity with letter symbols recording relationships but will need to further develop and deepen this.

Students often interpret an algebraic formula or equation as a set of instructions to be followed, or as a problem to be solved, rather than understanding the symbols as a representation of a relationship. It can be useful to give them opportunities to work between different representations, including language, symbolic and graphical representations, to compare and identify equality and then to see how this relationship is captured in each representation.

Students may feel uncomfortable leaving their ‘answer’ as an expression or equation, and so an error such as rewriting $7 + m$ as $7m$ might not simply be a lack of understanding of the conventions of algebra, or the relationship being recorded, but that the student believes that their answer should be a single number or term.

Students’ intuition to use the letter symbol as shorthand may also lead to errors. For example, when asked to write a formula connecting the number of days in a week and the number of weeks, many students may write $7d = w$ (maybe reading this as seven days equals one week) where the correct formula should be $7w = d$. Again, using specific language to describe the relationship in words can help raise awareness of this. Examples are given below:

***Example 1:*** *Describe how each of the following is represented in this bar model.*




a) $x + 2 = 10$
b) $10 – x = 2$
c) $10 – 2 = x$

*Example 1* uses the familiar representation of a bar model to draw attention to the different relationships that exist and ways that they can be written symbolically. Although students may have seen this sort of image before, the focus here is particularly on the equality demonstrated between the top and bottom bars.

***Example 2:*** *Write an expression to represent each of these relationships:*

a) *Two different numbers add to 10.*
b) *Two numbers are 10 apart on the number line.*
c) *Two different numbers are added together to make a third number.*
d) *Two of the same number are added together to make another number.*

In *Example 2,* language is used as another representation to access the structure and give meaning to the symbols. It is useful to encourage students to work in both directions: from the language to symbolic algebra and to also describe the symbolic algebra verbally. Part *b* offers several possible correct solutions ($a + 10 = b$, $m – 10 = n$, $p – q = 10$). It is helpful to compare these solutions and unpack why this one example has multiple solutions while the others have just one correct answer.