4MD–2 Manipulating the multiplicative relationship

Manipulate multiplication and division equations, and understand and apply the commutative property of multiplication.

4MD–2 Teaching guidance

Pupils will begin year 4 with an understanding of some of the individual concepts covered in this criterion, but they need to leave year 4 with a coherent understanding of multiplicative relationships, and how multiplication and division equations relate to the various multiplicative structures.

Pupils need to be able to apply the commutativite property of multiplication in 2 different ways. The first can be summarised as ‘1 interpretation, 2 equations’. Here pupils must understand that 2 different equations can correspond to one context, for example, 2 groups of 3 is equal to 6 can be represented by $2 \times 3 = 6$ and by $3 \times 2 = 6$. Spoken language can support this understanding.

Language focus

“2 groups of 3 is equal to 6.”

“3, two times is equal to 6.”

“2 groups of 3 is equal to 3, two times.”

The second way that pupils must understand commutativity, can be summarised as ‘one equation, two interpretations’. Here, pupils must understand that a single equation, such as $2 \times 7 = 14$, can be interpreted in two ways.

Language focus

“2 groups of 7 is equal to 14.”

“7 groups of 2 is equal to 14.”

“2 groups of 7 is equal to 7 groups of 2.”

Pupils should understand that both interpretations correspond to the same total quantity (product).

Language focus

“factor times factor is equal to product”

“The order of the factors does not alter the product.”

An array is an effective way to illustrate this.

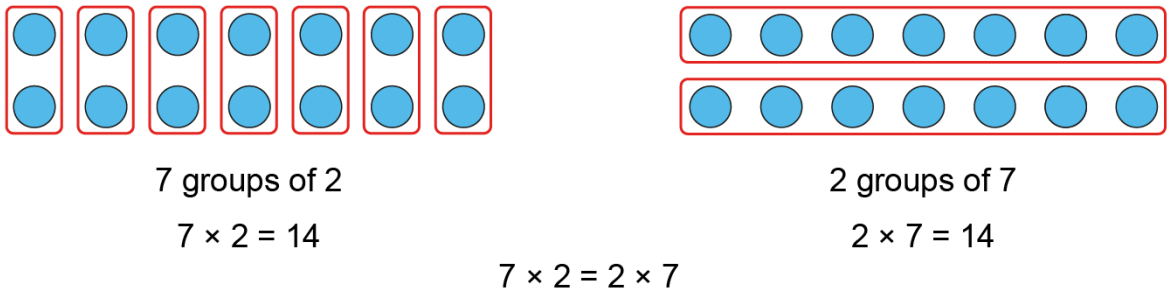


Figure 120: using an array to show that 7 groups of 2 and 2 groups of 7 both correspond to the same total quantity

Pupils must be able to describe what each number in the equation represents for the 2 different interpretations, in context.

$$7 \times 2 = 14$$

$$2 \times 7 = 14$$

Interpretation 1

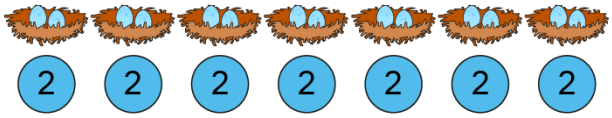


Figure 121: 7 groups of 2 – 7 nests of 2 eggs and seven 2-value counters

Language focus

"The 2 represents number of eggs in each nest/group".

"The 7 represents the number of nests/groups."

"The 14 represents the total number of eggs/product."

Interpretation 2

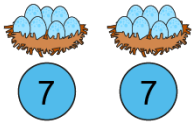


Figure 122: 2 groups of 7 – 2 nests of 7 eggs and two 7-value counters

Language focus

"The 2 represents the number of nests/groups."

"The 7 represents the number of eggs in each nest/group."

"The 14 represents the total number of eggs/product."

Pupils should be able to bring together these ideas to understand that either of a pair of multiplication equations can have two different interpretations.

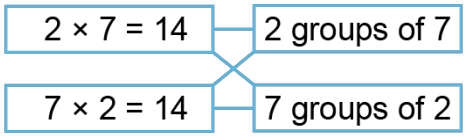


Figure 123: schematic diagram summarising the commutative property of multiplication and the different grouping interpretations

Pupils must understand that, because division is the inverse of multiplication, any multiplication equation can be rearranged to give division equations. The value of the product in the multiplication equation becomes the value of the dividend in the corresponding division equations.

$$2 \times 7 = 14 \qquad 7 \times 2 = 14 \qquad 14 \div 2 = 7 \qquad 14 \div 7 = 2$$

This means that the commutative property of multiplication has a related property for division.

Language focus

"If we swap the values of the divisor and quotient, the dividend remains the same."

As with multiplication, any division equation can be interpreted in two different ways, and these correspond to quotitive and partitive division.

$$14 \div 7 = 2$$

Partitive division

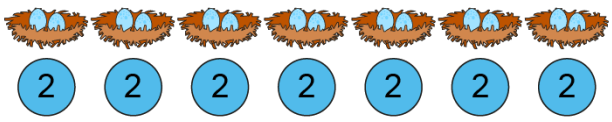


Figure 121: 7 groups of 2 – 7 nests of 2 eggs and seven 2-value counters

Language focus

"14 shared between 7 is equal to 2."

"The 14 represents the total number of eggs."

"The 7 represents the number of nests/shares."

"The 2 represents the number of eggs in each nest/share."

Quotitive division

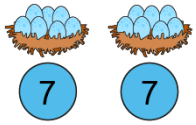


Figure 122: 2 groups of 7 – 2 nests of 7 eggs and two 7-value counters

Language focus

"14 divided into groups of 7 is equal to 2.

"The 14 represents the total number of eggs."

"The 7 represents the number of eggs in each nest/group."

"The 2 represents the number of nests/groups."

Pupils need to be able to fluently move between the different equations in a set, and understand how one known fact (such as $7 \times 2 = 14$) allows them to solve 4 different calculations each with two possible interpretations.

You can find out more about fluency in manipulating multiplication and division equations here in the calculation and fluency section: **4MD–2**

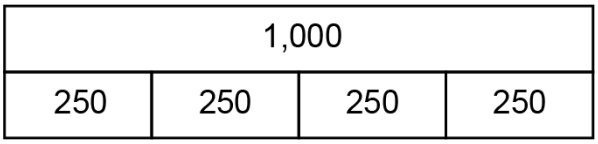
Making connections

Being able to move between the grouping and sharing structures of division supports calculation. For example, irrespective of the calculation context, pupils may find it more helpful to think of the sharing (partitive) structure when calculating $1{,}600 \div 2$ (1,600 shared or partitioned into 2 equal shares/parts is 800), rather than thinking about the grouping structure (800 twos in 1,600). Conversely, pupils may find it more helpful to think of $200 \div 25$ in terms of the grouping (quotitive) structure (how many groups of 25 there are in 200) rather than thinking about the sharing structure (200 shared or partitioned into 25 equal shares).

4MD–2 Example assessment questions

1. Using pictures of vases of flowers, draw two pictures which can be represented by the equation $5 \times 4 = 20$.
2. Write as many multiplication and division equations as you can to represent each picture.

   a.



   b.

3. Write a story that could be represented by this equation $3 \times 7 = 21$.
4. Using pictures of apples in bowls, draw 2 pictures which can be represented by the equation $18 \div 3 = 6$.
5. Use $15 \times 16 = 240$ to write 3 other related multiplication and division equations.
6. 45kg of animal feed is shared between some horses. They each get 5kg. How many horses were there?
7. 1m 40cm of ribbon was cut into equal pieces. Each piece is 14cm long. How many pieces of ribbon are there?
8. Fill in the missing numbers.

   $\square \div 20 = 5$ $\quad 3{,}000 \div \square = 250$ $\quad \square \times 100 = 5{,}400$